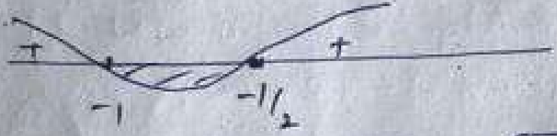
QID. 100401

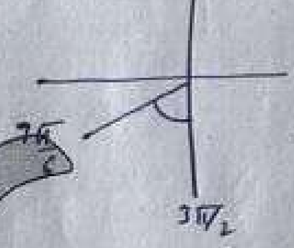
Solⁿ. $\cot^2\theta + 3\operatorname{cosec}\theta + 3 < 0$

$\dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{3}{\sin\theta} + 3 < 0 \Rightarrow 1 - \sin^2\theta + 3\sin\theta + 3(\sin^2\theta) < 0$

$2\sin^2\theta + 3\sin\theta + 1 < 0$

$(2\sin\theta + 1)(\sin\theta + 1) < 0$





$\theta \in (7\pi/6, 3\pi/2)$ $\boxed{\text{option B}}$

Q-ID: 100402

Solⁿ. $f(x) = \dfrac{x}{x-4}$, $g(x) = 4x+3$

given $(fog)^{-1}(\alpha) = 0 \Rightarrow \alpha = fog(0)$

Now $fog = f(g) = \dfrac{g}{g-4} = \dfrac{4x+3}{4x+3-4} = \dfrac{4x+3}{4x-1}$

$fog(0) = \dfrac{4\times0+3}{4\times0-1} = -3 \Rightarrow \alpha = -3$

$\dfrac{g(\alpha)}{f(\alpha)} = \dfrac{4\alpha+3}{\left(\frac{\alpha}{\alpha-4}\right)} \Rightarrow \dfrac{g(-3)}{f(-3)} = \dfrac{(-12+3)(-3-4)}{-3} = -21$ $\boxed{\text{option B}}$

Q-ID: 100403

Solⁿ. $(x-1)(x+1)(2x+1)(2x-3) = 15$

$(x-1)(2x+1)(x+1)(2x-3) - 15 = 0 \Rightarrow (2x^2-x-1)(2x^2-x-3) - 15 = 0$

let $y = 2x^2 - x$ we get $(y-1)(y-3) - 15 = 0$

$y^2 - 4y - 12 = 0 \Rightarrow y = -2, 6$

Now $2x^2 - x = -2 \Rightarrow$ we get $x = \frac{1}{4} + \frac{\sqrt{15}}{4}i, \ \frac{1}{4} - \frac{\sqrt{15}}{4}i$

$2x^2 - x = 6 \Rightarrow x = 2, -3/2$

Sum of modulus $= 2 + 3/2 + \sqrt{\frac{1}{16}+\frac{15}{16}} + \sqrt{\frac{1}{16}+\frac{15}{16}} = \frac{11}{2}$ $\boxed{\text{Option-C}}$

Q-ID: 100404

Sol^n $z = x+iy$, $\left|\frac{z-3i}{z+2i}\right| = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \frac{|x+iy-3i|}{|x+iy+2i|} = \frac{\sqrt{2}}{\sqrt{3}}$

$\cancel{\frac{\sqrt{(x-3)^2+y^2}}{\sqrt{x}}}$ $\quad \frac{\sqrt{x^2+(y-3)^2}}{\sqrt{x^2+(y+2)^2}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow$ squaring both the sides

we get $x^2+y^2-26y+19=0$ which is equation of circle whose centre is $(0,13)$ and $r = \sqrt{0+169-19} = \underline{5\sqrt{6}}$ $\boxed{\text{option: D}}$

Q-ID: 100405

Sol^n For S_1, given $-A^3 = (A^3)^t$, $B^2 = (B^2)^t$ and $AB = BA$

let $P = (AB)^6$

$P^t = ((AB)^6)^t = ((BA)^t)^6 = ((A^t)^3)^2 ((B^t)^2)^3 = ((-A^3))^2 (B^2)^3 = A^6B^6 = (AB)^6$

S_1 is true

Now, S_2, given $A^3 = (A^3)^t$, $-B^2 = (B^2)^t$ and $AB = BA$

let $P = (AB)^6$

$P^t = ((AB)^6)^t = ((AB)^t)^6 = (B^tA^t)^6 = ((B^t)^2)^3 ((A^t)^3)^2 = (-B^2)^3(A^3)^2$

$P^t = -B^6A^6 = -(BA)^6 = -(AB)^6 = -P \Rightarrow S_2$ is also true

$\boxed{\text{option: A}}$

Q-ID: 100406

Sol^n $|A| = \begin{vmatrix} 3 & -1 & 1 \\ 2 & -3 & \lambda \\ 1 & 1 & 3 \end{vmatrix}$ For no solution $|A| = 0 \Rightarrow \underline{\lambda = -4}$

also $|D_1| \neq 0 \Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ \mu & -3 & \lambda \\ -1 & 1 & 3 \end{vmatrix} \neq 0 \Rightarrow \underline{\mu \neq 3}$

$\boxed{\text{Option: A}}$

Q ID: 100407

Sol^n let $S = \underbrace{(a)+(a+d)+\cdots}_{11 \text{ terms of AP}} + \overbrace{(a+10d)+(a+10d)r+(a+10d)r^2+\cdots+(a+10d)r^{10}}^{11 \text{ term of GP}}$

Middle Term of AP = Middle Term of GP

6th Term of AP = 6th term of GP

$$a+5d = (a+10d).r^5$$

$d=2,\ r=\frac{1}{2}$ we get $a = -\frac{300}{31}$

Now $a_{11} = a+10d = +\frac{320}{31}$

$\boxed{\text{option A}}$

Q-ID: 100408

Sol^n $\lim_{x\to\infty} x\log_e\left(e\left(1+\frac{1}{x}\right)^{1-x}\right)$ let $x=\frac{1}{h} \Rightarrow h=\frac{1}{x}$, when $x\to\infty$, $h\to 0$

$$\lim_{h\to 0} \frac{1}{h}\log\left(e(1+h)^{1-1/h}\right)$$

$$\lim_{h\to 0} \frac{1}{h}\left[\log e + \left(1-\frac{1}{h}\right)\log(1+h)\right]$$

$$\lim_{h\to 0} \frac{1}{h}\left[1+\frac{h-1}{h}\log(1+h)\right] = \lim_{h\to 0} \frac{h+h\log(1+h)-\log(1+h)}{h^2}$$

$$\lim_{h\to 0} \frac{h-\log(1+h)}{h^2} + \frac{h\log(1+h)}{h^2}$$

$$\lim_{h\to 0} \frac{\cancel{h}-\left(\cancel{h}-\frac{h^2}{2}+\frac{h^3}{3}-\frac{h^4}{4}+\cdots\right)}{h^2} + \lim_{h\to 0}\frac{\log(1+h)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h^2}\left(-\frac{1}{2}+\frac{h}{3}+\frac{h^2}{4}+\cdots\right)}{\cancel{h^2}} + 1$$

$$\frac{1}{2}+1 = \frac{3}{2}$$

$\boxed{\text{Option: C}}$

Q-ID: 100409

Soln $y \times \sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$, on differentiation

$$y' \cdot \sqrt{x^2+1} + y \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \frac{1}{(\sqrt{x^2+1} - x)} \times \left[\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} - 1\right]$$

$$\frac{y'(x^2+1) + xy}{\cancel{\sqrt{x^2+1}}} = \frac{(x - \sqrt{x^2+1})}{(\sqrt{x^2+1} - x)\,\cancel{(\sqrt{x^2+1})}}$$

$$y'(x^2+1) + xy = -1$$

Again differentiating

$$y''(x^2+1) + y' \cdot 2x + xy' + y \cdot 1 = 0$$

$$y''(x^2+1) + 3xy' + y = 0$$

$\boxed{\text{option: B}}$

QID: 100410

Soln In S_1 we put $n=3$ we get $1.2 + 2.3 + 3.4 \le \frac{3 \times 49}{10}$

$20 \le 14.7$ (No True)

$\boxed{\text{option: D}}$

S_2 holds true for all values

QID: 100411

Soln. $\int_0^1 \tan^{-1}(1-x+x^2)\,dx = \int_0^1 \pi/2 - \cot^{-1}(1-x+x^2)\,dx$

$= \frac{\pi}{2} - \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = \frac{\pi}{2} - \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$

$= \frac{\pi}{2} - \int_0^1 (\tan^{-1}x + \tan^{-1}(1-x))\,dx$ but $\int_0^1 \tan^{-1}(1-x)\,dx = \int_0^1 \tan^{-1}(x)\,dx$ using property

$= \frac{\pi}{2} - 2\int_0^1 \tan^{-1}x\,dx$

$= \frac{\pi}{2} - 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1 = \underline{\log 2}$

$\boxed{\text{option-c}}$

QID: 100412

Sol^n $\frac{\bar{a}\cdot\bar{b}}{|b|} = \frac{4}{\sqrt5} \Rightarrow \frac{xy+2}{\sqrt{y^2+4}} = \frac{4}{\sqrt5}$

$\frac{\bar{a}\cdot\bar{c}}{|\bar{c}|} = \frac{3}{\sqrt5} \Rightarrow \frac{2+y}{\sqrt{y^2+4}} = \frac{3}{\sqrt5} \Rightarrow$ squaring both the sides

$y^2-5y+4=0 \Rightarrow y=1,4$

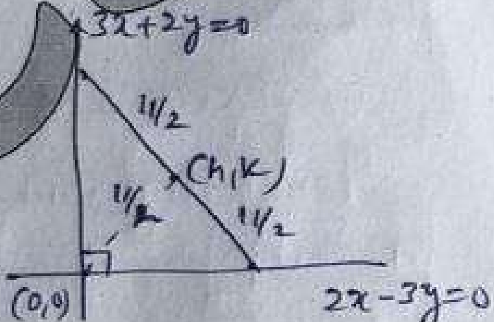
$\boxed{y=1 \Rightarrow x=2}$ ✓

$y=4 \Rightarrow x=3/2$ As $|c|<3$ this is discarded

Now. $\bar{a}\cdot(\bar{b}\times\bar{c}) = \begin{vmatrix} x & 1 & 1 \\ y & 0 & 2 \\ 0 & 2 & y \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix} = -7$ $\boxed{\text{option A}}$

QID: 100413

Soln let the mid point of the rod be (h,k). In a right triangle mid-point of hypotenuse is equidistant from three vertices so



$(h-0)^2+(k-0)^2=\left(\frac{11}{2}\right)^2$

$h^2+k^2=\frac{121}{4} \Rightarrow x^2+y^2=\left(\frac{11}{2}\right)^2$ $\boxed{\text{option-B}}$

QID: 100414

Sol^n. $\frac{dy}{dx} = \frac{e^{2y}+x^2}{x^3} \Rightarrow e^{-2y}\frac{dy}{dx} - e^{-2y}\cdot\frac{1}{x} = \frac{1}{x^3}$; let $e^{-2y}=t$

$-2e^{-2y}\cdot\frac{dy}{dx} = \frac{dt}{dx}$

Eq^n becomes $\frac{dt}{dx}+\frac{2}{x}t = -\frac{2}{x^3}$, $P=\frac{2}{x}$, $Q=-\frac{2}{x^3}$

$IF = e^{\int\frac{2}{x}dx} = x^2$

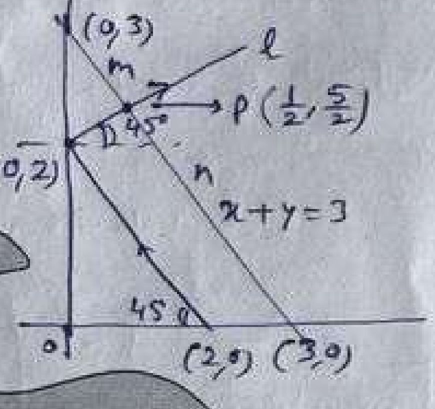
Sol^n: $t\cdot x^2 = \int x^2\cdot -\frac{2}{x^3}dx + c \Rightarrow \underline{e^{-2y}\cdot x^2 = -2\log x + c}$

given $y(e)=1$, $\Rightarrow x=e, y=1$ we get $c=3$

$e^{-2y}.x^2 = -2\log x + 3$, Now put $x=1, \Rightarrow y=\log\frac{1}{\sqrt{3}}$ $\boxed{\text{Option D}}$

QID: 100415

Sol^n



Slope of $l = \tan 45° = 1$

Eq^n of $l \Rightarrow y-2 = 1(x-0)$

$y = x+2$

Solving $y = x+2$ & $x+y=3$ we get

$P\left(\frac{1}{2}, \frac{5}{2}\right)$

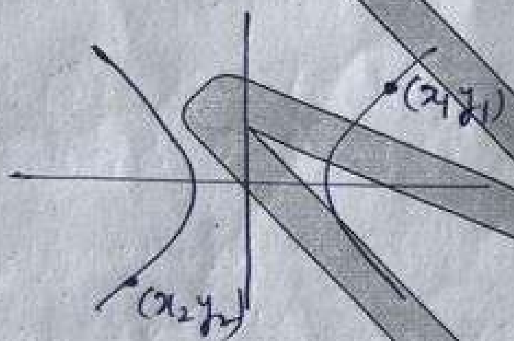
Using section formula

$\frac{1}{2} = \frac{3m + 0\times n}{m+n} \Rightarrow n = 5m$ sub. in $\frac{2n-m}{2n+m} = \frac{10m-m}{10m+m} = \frac{9}{11}$

$\boxed{\text{Option -A}}$

QID: 100416

Sol^n



Slope of line, $y = 2x \Rightarrow m = 2$

differentiating $x^2 - y^2 = 60$

$2x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x_1}{y_1}$

$2 = \frac{x_1}{y_1} \Rightarrow x_1 = 2y_1$ sub. in hyperbola

$x_1^2 - y_1^2 = 60$

$4y_1^2 - y_1^2 = 60 \Rightarrow y_1^2 = 20 \Rightarrow y_1 = \pm\sqrt{20}$, $x_1 = \pm 2\sqrt{20}$

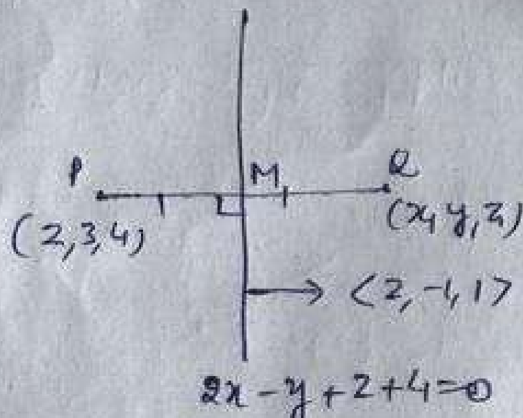
Pts are $(2\sqrt{20}, \sqrt{20})$ & $(-2\sqrt{20}, -\sqrt{20})$

$d = \sqrt{(2\sqrt{20}+2\sqrt{20})^2 + (\sqrt{20}+\sqrt{20})^2}$

$d = \sqrt{16\times 20 + 4\times 20} = 20$ $\boxed{\text{option: B}}$

Q-ID: 100417

Sol^n Dr's of the normal to the plane will be dr's of the line PQ



Eq of PQ $\frac{x-2}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$

Any point M $(2\lambda+2, -\lambda+3, \lambda+4)$

M lies on the plane $2x-y+z+4=0$

$2(2\lambda+2) - (-\lambda+3) + (\lambda+4) + 4 = 0 \Rightarrow \lambda = -3/2$

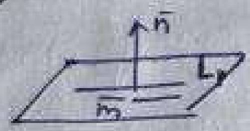
M $(-1, 9/2, 5/2)$ using mid pt $-1 = \frac{x_1+2}{2}, \frac{9}{2} = \frac{3+y_1}{2}, \frac{5}{2} = \frac{4+z_1}{2}$

we get Q $(-4, 6, 1)$ which lies on $\frac{x-6}{5} = y-8 = \frac{z-5}{2}$, $\boxed{\text{option-c}}$

QID: 100418

Sol^n $L_1 = \frac{x-1}{2} = \frac{y-1}{p} = \frac{z-2}{2}$ lies on the plane $2x+3y-4z=3$

so. $\bar{m} \cdot \bar{n} = 0$



$2\times2 + p\times3 + 2\times-4 = 0 \Rightarrow p = 4/3$

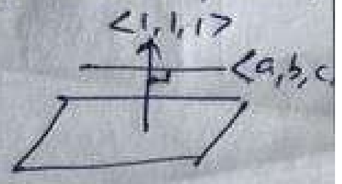
If dr' of L_2 are $<a, b, c>$ then.

$L_2 = \frac{x-1}{a} = \frac{y-2}{b} = \frac{z-0}{c}$

L_1 and L_2 intersect $\Rightarrow \begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$$\begin{vmatrix} 1-1 & 1-2 & 2-0 \\ 2 & 4/3 & 2 \\ a & b & c \end{vmatrix} = 0 \Rightarrow \boxed{7a-6b-3b=0}$$

Also L_2 is $\parallel$ to plane $x+y+z=1$



$a+b+c=0$

$7a-6b-3b=0$

$$\frac{a}{-6+3} = \frac{-b}{7+3} = \frac{c}{7+6}$$ $<a, b, c> = <-3, -10, 13>$

Eqⁿ. of L_2 : $\frac{x-1}{-3} = \frac{y-2}{-10} = \frac{z-0}{13}$

As $(1,2,0)$ lies on $\frac{8x-5}{-3} = \frac{4y-3}{-5} = \frac{8z-13}{13}$ $\boxed{\text{option-A}}$

$\frac{x-5/8}{-3/8} = \frac{y-3/4}{-5/4} = \frac{z-13/8}{13/8}$ has dr's $\langle -3, -10, 13 \rangle$

QID: 100419

Solⁿ. $2, 6, 12, 8, k, 20$, $\bar{x} = 12$, $n = 6$

$\bar{x} = \frac{\sum x_i}{n} \Rightarrow 12 = \frac{2+6+12+8+k+20}{6} \Rightarrow k = 24$

median of $2, 6, 8, 12, 20, 24$ is $\frac{8+12}{2} = 10$

$M.D = \frac{|x_i - M_d|}{n} \Rightarrow m = \frac{|2-10| + |6-10| + |8-10| + |12-10| + |20-10| + |24-10|}{6}$

$\boxed{m = \frac{20}{3}}$

Now, $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n^2} = \frac{(2-12)^2 + (6-10)^2 + (8-12)^2 + 0 + (20-12)^2 + (24-12)^2}{6}$

$\sigma^2 = 60 \Rightarrow \frac{\sigma^2}{m} = \frac{60}{20/3} = 9$ $\boxed{\text{Ans: A}}$

QID: 100420

Solⁿ.

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$\sim q \to q$	$(p \to \sim p) \wedge (\sim q \to q)$	$\sim [(p \to \sim p) \wedge (\sim q \to q)]$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	F	T
F	T	T	F	T	T	T	F
F	F	T	T	T	F	F	T

$\boxed{\text{option-C}}$

QID: 100421

Solⁿ. $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + {}^{20}C_3 x^3 + \dots + {}^{20}C_{20} x^{20}$.

$(x-1)^{20} = {}^{20}C_0 x^{20} - {}^{20}C_1 x^{19} + {}^{20}C_2 x^{18} - {}^{20}C_3 x^{17} + \dots + {}^{20}C_{20}$.

$(x^2-1)^{20} = \left({}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \cdots + {}^{20}C_{20} x^{20}\right)\left({}^{20}C_0 x^{20} - {}^{20}C_1 x^{19} + {}^{20}C_2 x^{18} - \cdots + {}^{20}C_{20}\right)$ —— ①

$\longrightarrow T_{r+1} = {}^{20}C_r \cdot (x^2)^r = {}^{20}C_r \cdot (-1)^r \cdot x^{2r}$

Now $\sum_{k=1}^{20} {}^{20}C_k \cdot {}^{20}C_{k-1} (-1)^k = -{}^{20}C_1 \cdot {}^{20}C_0 + {}^{20}C_2\, {}^{20}C_1 - {}^{20}C_3\, {}^{20}C_2 + \cdots + {}^{20}C_{20} \cdot {}^{20}C_{19}$

= It is coefficient of x^{19} in ①

= but there is No term containing x^{19} in $(x^2-1)^{20}$

So $\sum_{k=1}^{20} {}^{20}C_k \cdot {}^{20}C_{k-1} \cdot (-1)^k = \underline{\underline{0}}$

and

$\sum_{k=0}^{20} ({}^{20}C_k)^2 (-1)^k = ({}^{20}C_0)^2 - ({}^{20}C_1)^2 + ({}^{20}C_2)^2 - ({}^{20}C_3)^2 + \cdots ({}^{20}C_{20})^2$

= Coefficient of x^{20} in ①

= Put $r=10$ in $T_{r+1} = {}^{20}C_{10} (-1)^{10} \cdot x^{20}$

$= \underline{\underline{{}^{20}C_{10}}}$

<u>Also</u>

$(1+x)^{21} = {}^{21}C_0 + {}^{21}C_1 x + {}^{21}C_2 x^2 + \cdots + {}^{21}C_{21} x^{21}$

$(x-1)^{21} = {}^{21}C_0 x^{21} - {}^{21}C_1 x^{20} + {}^{21}C_2 x^{19} - \cdots - {}^{21}C_{21}$.

$(1+x)^{21}(x-1)^{21} = \left({}^{21}C_0 + {}^{21}C_1 x + {}^{21}C_2 x^2 + \cdots + {}^{21}C_{21} x^{21}\right) \times \left({}^{21}C_0 x^{21} - {}^{21}C_1 x^{20} + {}^{21}C_2 x^{19} - \cdots - {}^{21}C_{21}\right)$

$\downarrow$

$(x^2-1)^{21} \Rightarrow T_{r+1} = -{}^{21}C_r (-x^2)^r = -{}^{21}C_r \cdot (-1)^r \cdot x^{2r}$

$\sum_{k=0}^{21} ({}^{21}C_k)^2 (-1)^k = ({}^{21}C_0)^2 - ({}^{21}C_1)^2 + ({}^{21}C_2)^2 - \cdots$

= Coefficient of x^{21} in $(x^2-1)^{21}$ but there is No term containing x^{21} in $T_{r+1} = -{}^{21}C_r (-1)^r (x)^{2r}$

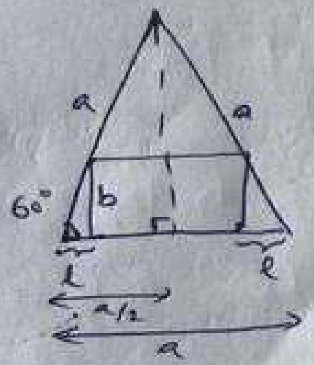
$= \underline{0}$

So $0 + {}^{20}C_{10} + 0 = p \cdot {}^{19}C_{10} \Rightarrow \frac{20!}{10!\,10!} = p \times \frac{19!}{10!\,9!} \Rightarrow \boxed{p = 2}$ Ans

Q ID: 100422

Sol^n.

$\tan 60 = \frac{b}{l} \Rightarrow b = \sqrt{3}l$



length of rectangle $= 2\left(\frac{a}{2} - l\right) = a - 2l$

Area, $A = (a-2l)b$; $b = \sqrt{3}l$

$A = (a-2l)\sqrt{3}l = \sqrt{3}al - 2\sqrt{3}l^2$

$\frac{dA}{dl} = \sqrt{3}a - 4\sqrt{3}l \Rightarrow \frac{dA}{dl} = 0 \Rightarrow l = a/4$

$A_{max} = \left(a - \frac{a}{2}\right) \times \sqrt{3} \times \frac{a}{4} = \frac{\sqrt{3}a^2}{8} \Rightarrow \frac{\sqrt{3}a^2}{8} = \frac{25\sqrt{3}}{2} \Rightarrow a^2 = 100$

Perimeter of triangle $= 3 \times a = 3 \times 10 = 30$ $\boxed{\text{Ans: } 30}$

Q-ID: 100423

Sol^n.

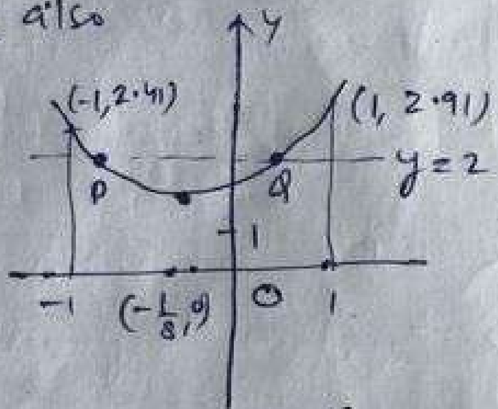
$$f(x) = \begin{cases} x^2 + 2x + 2, & x \le -1 \\ \left[x^2 + \frac{x}{4} + \frac{5}{3}\right], & -1 < x < 1 \\ x^2 - 2x + 4, & x \ge 1 \end{cases}$$

at $x = -1$, $f(-1) = 1 - 2 + 2 = 1$

$RHL = \lim_{x \to -1+h} \left[x^2 + \frac{x}{4} + \frac{5}{3}\right] = \lim_{h \to 0} \left[(-1+h)^2 + \frac{-1+h}{4} + \frac{5}{3}\right]$

$= \lim_{h \to 0} \left[+1 - 2h + h^2 - \frac{1}{4} + \frac{h}{4} + \frac{5}{3}\right] = 2$, Discontinuous at $x = -1$

Similarly discontinuous at $x = 1$ also



Greatest integer function is discontinuous at integral values, $x^2 + \frac{x}{4} + \frac{5}{3}$ will be equal '2' at two values of x in the interval $(-1, 1)$

$x^2 + x/4 + 5/3 \in [1.65, \infty)$

Thus $f(x)$ is discontinuous at $-1, 1, P$ & Q $\boxed{4 \text{ points}}$

Q ID: 100424

Solⁿ. When a number is divisible by '11' then the difference of the sum of digits at Even and odd places should be '0' or multiple of 11.

'0' is NOT possible as total sum of digits is = 27

The only possibility is (5, 6, 8) and (1, 3, 4) [19 − 8 = 11]

(5, 6, 8) ↓ odd place OR even place; (1, 3, 4) ↓ even place OR odd place

$2((3\times2\times1)\times(3\times2\times1)) = \underline{\underline{72}}$ $\boxed{\text{Ans: }72}$

Q ID: 100425

Solⁿ. Total no. of ordered pairs = 90 (As m ≠ n)

ordered pairs satisfying a+b = c+d

sum = 3, (1,2),(2,1) = 2 cases

sum = 4, (1,3)(3,1) = 2

sum = 5, (1,4)(3,2)(2,3)(4,1) = 4

sum = 6, (1,5)(5,1)(2,4)(4,2) = 4

sum = 7, (1,6)(2,5)(3,4)(4,3)(5,2)(6,1) = 6

sum = 8, (1,7)(2,6)(3,5)(5,3)(6,2)(7,1) = 6 → sum = 9 ⇒ 8 cases

sum = 10, (1,9)(2,8)(3,7)(4,6)(6,4)(7,3)(8,2)(9,1) = 8

sum = 11, (1,10)(2,9)(3,8)(4,7)(5,6)(6,5)(7,4)(8,3)(9,2)(10,1) ⇒ 10

sum = 12, (2,10)(3,9)(4,8)(5,6)(6,5)((8,4)(9,3)(10,2) = 8

sum = 13, (3,10)(4,9)(5,8)(6,7)(7,6)(8,5)(9,4)(10,3) ⇒ 8

sum = 14, (4,10)(5,9)(6,8)(8,6)(9,5)(10,4) ⇒ 6

sum = 15, (5,10)(6,9)(7,8)(8,7)(9,6)(10,5) ⇒ 6

sum = 16, (6,10)(7,9)(9,7)(10,6) ⇒ 4

sum = 17, (7,10)(8,9)(9,8)(10,7) ⇒ 4

sum = 18, (8,10)(10,8) ⇒ 2

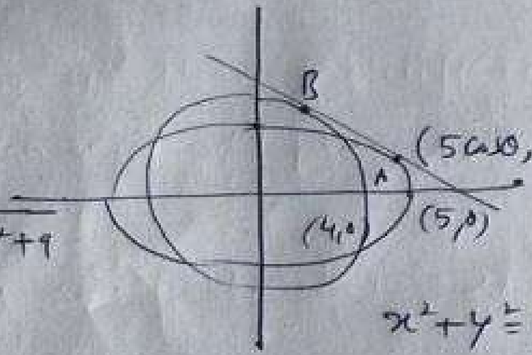
sum = 19, (10,9)(9,10) ⇒ 2

$$p = \frac{4[2\times2+4\times4+6\times6+8\times8]+10\times10}{90\times90} = \frac{580}{90\times90} = \frac{145}{45\times45} \Rightarrow (45)^2 p = \underline{145}$$

$\boxed{145}$

QID: 100426

Sol^n. Let the common tangent to the circle by $y = mx + 4\sqrt{1+m^2}$
and to the ellipse $y = mx + \sqrt{25m^2+9}$



$x^2 + y^2 = 16$
$r = 4$

ie $4\sqrt{1+m^2} = \sqrt{25m^2+9}$

$m = \frac{\sqrt{7}}{3} \Rightarrow y = \frac{\sqrt{7}}{3}x + \frac{16}{3}$ — ①

$\frac{x^2}{25} + \frac{y^2}{9} = 1$
$a^2 = 25,\ b^2 = 16$

Let point of contact 'A' on the ellipse be $(5\cos\theta, 3\sin\theta)$

then eq^n of tangent at this point on ellipse is $\frac{x}{25}.5\cos\theta + \frac{y}{9}.3\sin\theta = 1$ — ②

Comparing ① and ②

$\sqrt{7}x - 3y + 16 = 0$ & $\frac{x}{5}\cos\theta + \frac{y}{3}\sin\theta - 1 = 0$

$\frac{\frac{\cos\theta}{5}}{\sqrt{7}} = \frac{\frac{\sin\theta}{3}}{-3} = \frac{-1}{16}$ we get $\cos\theta = -\frac{5\sqrt{7}}{16}$, $\sin\theta = \frac{9}{16}$

$A\left(-\frac{25\sqrt{7}}{16}, \frac{27}{16}\right)$ length of Tangent $= \sqrt{S_1}$ (to the circle)

$= \sqrt{\left(-\frac{25\sqrt{7}}{16}\right)^2 + \left(\frac{27}{16}\right)^2 - 1} \Rightarrow L = \frac{\sqrt{1008}}{16}$

Now, $32L^2 = 32 \times \frac{1008}{16 \times 16} = \underline{126}$ $\boxed{126}$

QID. 100427

Sol^n. $f_n(x) = \sum_{j=1}^{n} \cot^{-1}\left(1 - (x+j) + (x+j)^2\right)$, $x \geq 0$ then $\sum_{j=1}^{10} (j^2+1)\sin^2(f_j(0))$

$f_n(x) = \sum \tan^{-1}\left[\frac{1}{1 + (x+j)(x+j-1)}\right]$ taking $(x+j)$ common, & $\cot^{-1}x = \tan^{-1}\frac{1}{x}$

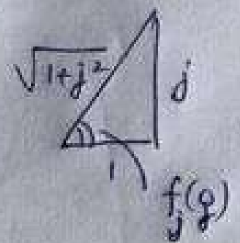
$= \sum \tan^{-1}\left[\frac{(x+j) - (x+j-1)}{1 + (x+j)(x+j-1)}\right]$

$= \sum_{j=1}^{n}\left[\tan^{-1}(x+j) - \tan^{-1}(x+j-1)\right] = \sum_{j=1}^{n} 1$

$$f_n(x) = \left(\tan^{-1}(x+1) - \tan^{-1}(x)\right) + \left(\tan^{-1}(x+2) - \tan^{-1}(x+1)\right) + \cdots + \left(\tan^{-1}(x+n) - \tan^{-1}(x+n-1)\right)$$

$$f_n(x) = \tan^{-1}(x+n) - \tan^{-1}x = \tan^{-1}\left(\frac{n}{1+x(x+n)}\right) \Rightarrow f_n(0) = \tan^{-1}(n)$$

$$\tan(f_n(0)) = n \Rightarrow \tan(f_j(0)) = \frac{j}{1}$$



$$\sin(f_j(0)) = \frac{j}{\sqrt{1+j^2}}$$

$$(1+j^2)\sin^2(f_j(0)) = j^2$$

Now

$$\sum_{j=1}^{10} (j^2+1)\sin^2(f_j(0)) = \sum_{j=1}^{10} j^2 = (1^2+2^2+\cdots+10^2)$$

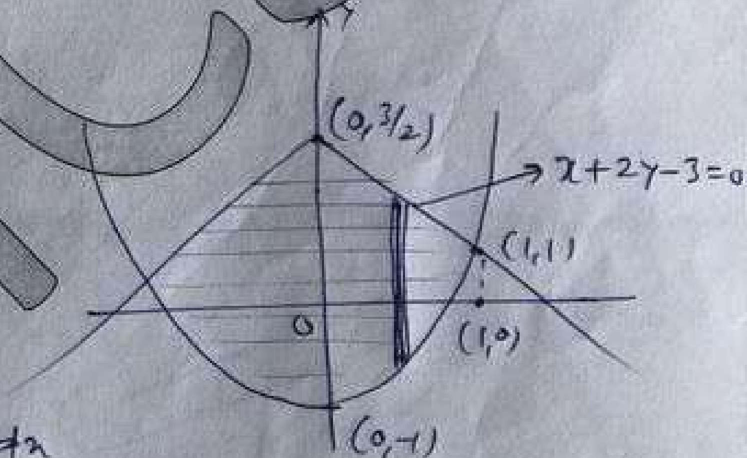
$$= \frac{10\times(11)\times(21)}{6} = \underline{\underline{385}}$$

$$\boxed{385}$$

QID 400428

$y = 2x^2 - 1, \quad |x| = 3 - 2y$

Solving the two eqn we get $x = 1, y = 1$



$$A = 2\int_0^1 \left(\frac{3-x}{2}\right) - (2x^2-1)\, dx$$

$$= 2\int_0^1 -2x^2 - \frac{x}{2} + \frac{5}{2}\, dx$$

$$= \int_0^1 -4x^2 - x + 5\, dx$$

$$= \left[-\frac{4x^3}{3} - \frac{x^2}{2} + 5x\right]_0^1 = \left[-\frac{4}{3} - \frac{1}{2} + 5\right]$$

$$A = \frac{19}{6} \Rightarrow 12A = \underline{\underline{38}}$$

$$\boxed{38}$$

QID 100429

Soln. Roots of $x^2+(\sqrt{3}-\sqrt{2}-1)+(\sqrt{3}-2-\sqrt{6}+2\sqrt{2})=0$ are $\tan A/_2$ $\tan B/_2$

So $\tan A/_2+\tan B/_2=-(\sqrt{3}-\sqrt{2}-1)=\sqrt{2}+1-\sqrt{3}$

$\tan A/_2 \tan B/_2=\sqrt{3}-2-\sqrt{6}+2\sqrt{2}$

$$\tan\left(A/_2+B/_2\right)=\frac{\tan A/_2+\tan B/_2}{1-\tan A/_2\tan B/_2}=\frac{\sqrt{2}+1-\sqrt{3}}{1-(\sqrt{3}-2-\sqrt{6}+2\sqrt{2})}$$

$$=\frac{\sqrt{2}+1-\sqrt{3}}{3-2\sqrt{2}+\sqrt{6}-\sqrt{3}}=\frac{\sqrt{2}+1-\sqrt{3}}{(\sqrt{2}-1)^2+\sqrt{3}(\sqrt{2}-1)}$$

$$=\frac{\sqrt{2}+1-\sqrt{3}}{(\sqrt{2}-1)(\sqrt{2}-1+\sqrt{3})}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$=\frac{2+\sqrt{2}-\sqrt{6}+\sqrt{2}+1-\sqrt{3}}{(2-1)(\sqrt{2}-1+\sqrt{3})}$$

$$=\frac{3+2\sqrt{2}-\sqrt{6}-\sqrt{3}}{(\sqrt{2}-1+\sqrt{3})}$$

On simplification further we get $\tan\left(\frac{A+B}{2}\right)=\sqrt{6}-\sqrt{3}+\sqrt{2}-2$
$\downarrow$
$\tan(15/2)$

which gives $\frac{A+B}{2}=\frac{15}{2} \Rightarrow A+B=15^\circ$

Now $12\sec^2(A+B)=12\sec^2 60=12\times 4=\underline{\underline{48}}$ $\boxed{48}$

QID 100430

Soln. The value of $2\int_{-1}^{4}|x-3|+[x]\,dx$

$$2\left[\int_{-1}^{3}-(x-3)dx+\int_{3}^{4}(x-3)dx\right]+2\left[\int_{-1}^{0}-1\,dx+\int_{0}^{1}0\,dx+\int_{1}^{2}1\,dx+\int_{2}^{3}2\,dx+\int_{3}^{4}3\,dx\right]$$

$\left\{\begin{matrix}x-3=0\\x=3\end{matrix}\right\}$

$$2\left[\left(-\frac{x^2}{2}+3x\right)_{-1}^{3} + \left(\frac{x^2}{2}-3x\right)_{3}^{4}\right] + 2\left[(-x)_{-1}^{0} + (x)_{1}^{2} + (2x)_{2}^{3} + (3x)_{3}^{4}\right]$$

$$2\left[\left\{\left(-\frac{9}{2}+9\right)-\left(-\frac{1}{2}-3\right)\right\}+\left\{\left(\frac{16}{2}-12\right)-\left(\frac{9}{2}-9\right)\right\}\right]+2\left[-(0+1)+(2-1)+(6-4)+(12-9)\right]$$

$$2\left[\left(\frac{1}{2}+\frac{7}{2}\right)+\left(-4+\frac{9}{2}\right)\right]+2\left[-1+1+2+3\right]$$

$$2\left[8+\frac{1}{2}\right]+2\left[5\right]=16+1+10=27 \qquad \boxed{27}$$