Solutions of Maths JEE-paper-2, June 2022 (Slot-2)

ID: 121 The equation of the plane passing through the intersection of the planes $\bar{r}.(i+2j-k)=3$ and $\bar{r}.(2i-j+3k)=2$, and parallel to the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{1}$, is

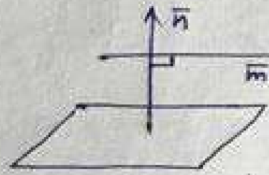
(a) $\bar{r}.(-5i+10j-15k)=4$ (b) $\bar{r}.(-5i+10j-15k)=1$

(c) $\bar{r}.(-9i+6j-3k)=4$ (d) $\bar{r}.(-9i+6j-3k)=1$

Sol^n.

$x+2y-z-3=0$, $2x-y+3z-2=0$

$(x+2y-z-3)+\lambda(2x-y+3z-2)=0$ ——— (i)



$x(1+2\lambda)+y(2-\lambda)+z(-1+3\lambda)-3-2\lambda=0$

dr' of the plane $\langle 1+2\lambda, 2-\lambda, -1+3\lambda\rangle$

dr' of the line $\langle 1, 2, 1\rangle$

using $\bar{m}.\bar{n}=0 \Rightarrow 1(1+2\lambda)+2(2-\lambda)+1(-1+3\lambda)=0$ $\underline{\lambda=-4/3}$

substituting in (i) $\Rightarrow -5x+10y-15z-1=0$

$\boxed{\bar{r}.(-5i+10j-15k)-1=0}$ $\boxed{\text{option-B}}$

I.D.-122 let $f, g: R\to R$ be functions defined by $f(x)=x-7$ and $g(x)=[7+\sin x]$ where $[t]$ is the greatest integer less than or equal to t. Then the number of points in $[0,\pi]$ where the function $fog+gof$ is not continuous, is

(a) 1 (b) 2 (c) 3 (d) 5

Sol^n.

$fog=f(g)=g(x)-7=[7+\sin x]-7=7+[\sin x]-7=[\sin x]$

$gof=g(f)=[7+\sin f(x)]=[7+\sin(x-7)]=7+[\sin(x-7)]$

$fog+gof=[\sin x]+7+[\sin(x-7)]$ in $[0,\pi]$

The greatest integer function is NOT continuous at integral values, $\sin x$ will be integer at $x=0, \pi/2$ and π so the function will NOT be continuous at 3 pts. $\boxed{\text{option-C}}$

ID-123 let m and n be non-negative integers such that for $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ $\tan x + \sin x = m$, $\tan x - \sin x = n$ then the possible ordered pair (m,n) is

(a) (2,1) but not (3,4) (b) (3,4) but not (2,1)

(c) both (2,1) and (3,4) (d) neither (2,1) nor (3,4)

Sol^n Adding and subtracting $\tan x + \sin x = m$, $\tan x - \sin x = n$

$2\tan x = m+n$, $2\sin x = m-n$

$\cos x = \frac{m-n}{m+n}$ If $m=3, n=4 \Rightarrow \cos x = -\frac{1}{4}$ but $\cos x = +ve$ in $(-\frac{\pi}{2}, \frac{\pi}{2})$

so (3,4) is NOT possible

If $(m,n) = (2,1)$ then $\sin x = \frac{m-n}{2} = \frac{1}{2}$ and $\cos x = \frac{1}{3}$ (Not possible)

$\boxed{\text{option-D}}$

ID-124 let $f(x) = (x+4)^2 - 4$, $x \geq -4$. Then $\{x : f(x) = f^{-1}(x)\}$ is equal to

(a) $\{-4,-3,3,4\}$ (b) $\{-3,0,4\}$ (c) $\{-4,3\}$ (d) $\{-4,-3\}$

Sol^n. As function is inverse of itself $f(x) = x \Rightarrow x = (x+4)^2 - 4$

$(x+4)^2 - (x+4) = 0 \Rightarrow (x+4)(x+4-1) = 0$, $(x+4)(x+3) = 0 \Rightarrow x = -3, -4$

$\boxed{\text{option-D}}$

ID-125 let z be a complex number and $\theta = \tan^{-1}\left(\left|\frac{Im(z)}{Re(z)}\right|\right)$ be an acute angle. If $\arg(z) = \theta - \pi$, $|Re(z)| = |Re(1-2i)^{-3}|$ and $|Im(z)| = |Im(1-2i)^{-3}|$, then $125\, Im\left(z + \frac{2i}{z}\right)$ is equal to

(a) -2752 (b) -1377 (c) -1152 (d) -627

Sol^n. As $\arg(z) = \theta - \pi$, complex number lies in the 3rd Quadrant ie x, y are $\underline{-ve}$

$|Re(z)| = |Re(1-2i)^{-3}| = \left|Re\left(\frac{1}{(1-2i)^3}\right)\right|$

$|x| = \left|Re\left(\frac{(1+2i)^3}{(1+4)^3}\right)\right| = \left|\frac{1}{125} Re(-11-2i)\right| = \frac{11}{125} \Rightarrow \boxed{x = -\frac{11}{125}}$

$|Im(z)| = |Im(1-2i)^{-3}| = \left|\frac{Im(-11-2i)}{125}\right| = -\frac{2}{125} \Rightarrow \boxed{y = \frac{-2}{125}}$

$z = (-11-2i)/125 \Rightarrow |z|^2 = 125/125 = 1$

Now $125\,\mathrm{Im}\left(z+\frac{2i}{\bar{z}}\right) = 125\,\mathrm{Im}\left(z+\frac{2iz}{z\bar{z}}\right) \Rightarrow 125\,\mathrm{Im}\left(\frac{-11-2i}{125}+\frac{2i(-11-2i)}{125}\right)$

$125\left(\frac{-2}{125}-22\right) = -2-125\times 22 = \underline{-2752}$ $\boxed{\text{option-a}}$

ID-126 Let $[a_{ij}]=A$, $\det(A)\neq 0$, and $B=[b_{ij}]$ be two 3×3 matrices. If $b_{ij}=3^{i-j}a_{ij}$ for all $i,j=1,2,3$ then

(a) $27\det(A)=\det(B)$
(b) $27\det(A)=\det(B)$
(c) $\det(A)=\det B$
(d) $\det(A)=27\det(B)$

Soln

$$\det(B)=\begin{vmatrix} b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{vmatrix}$$

$b_{ij}=3^{i-j}a_{ij}$

$$=\begin{vmatrix} a_{11} & \frac{1}{3}a_{12} & \frac{1}{9}a_{13}\\ 3a_{21} & a_{22} & \frac{1}{3}a_{23}\\ 9a_{31} & 3a_{32} & a_{33}\end{vmatrix} = \frac{1}{9\times3}\begin{vmatrix} 9a_{11} & 3a_{12} & a_{13}\\ 9a_{21} & 3a_{22} & a_{23}\\ 9a_{31} & 3a_{32} & a_{33}\end{vmatrix}$$

$$\det(B)=\frac{9\times3}{27}\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=\det(A)$$

$\boxed{\text{option-c}}$

ID-127 Let A be a 3×3 symmetric matrix with integer entries. If the sum of all diagonal elements of A^2 is 2, then the total number of such matrices A is equal to

(a) 12 (b) 6 (c) 18 (d) 24

Soln A is symmetric $A=\begin{bmatrix} a & b & c\\ b & d & f\\ c & f & e\end{bmatrix}$

$$A^2=\begin{bmatrix} a^2+b^2+c^2 & & \\ & b^2+d^2+f^2 & \\ & & c^2+f^2+e^2\end{bmatrix}$$

ATQ $\Rightarrow$ $a^2+b^2+c^2+b^2+d^2+f^2+c^2+f^2+e^2=2$

$a^2+d^2+e^2+2(b^2+c^2+f^2)=2$

a, b, c, d, e, f are integers then

$a=d=\pm1$, rest $=0$
$a=e=\pm1$ " $=0$
$d=e=\pm1$ " $=0$

$\Big\}$ 2×6 cases $=12$ cases

Also $b=\pm1$, rest $=0$
$c=\pm1$, rest $=0$
$f=\pm1$, rest $=0$

$\Big\}$ 6 case Total $=12+6=18$

$\boxed{\text{Option C}}$

ID-128 If $({}^{20}C_1)^2+2({}^{20}C_2)^2+3({}^{20}C_3)^2+\dots+20({}^{20}C_{20})^2=K$ then $\frac{(20!)^2K}{40!}$

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) 5 (d) 10

Soln

$$\sum_{r=1}^{r=20} r({}^{20}C_r)^2=\sum r.{}^{20}C_r.{}^{20}C_r$$

$$=\sum r.\frac{20}{r}\,{}^{19}C_{r-1}\,{}^{20}C_r \quad \left(\text{using } {}^nC_r=\frac{n}{r}\,{}^{n-1}C_{r-1}\right)$$

$$=20\sum_{r=1}^{20}{}^{19}C_{r-1}\,{}^{20}C_r=20\left[{}^{19}C_0\,{}^{20}C_1+{}^{19}C_1.{}^{20}C_2+\cdots\right]$$

Now $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x^1+{}^{20}C_2x^2+{}^{20}C_3x^3+\dots+{}^{20}C_{20}x^{20}$

also $(x+1)^{19}={}^{19}C_0x^{19}+{}^{19}C_1x^{18}+{}^{19}C_2x^{17}+\dots+{}^{19}C_{19}$.

$(1+x)^{20}.(x+1)^{19}=({}^{20}C_0+{}^{20}C_1x^1+{}^{20}C_2x^2+\dots+{}^{20}C_{20}x^{20})({}^{19}C_0x^{19}+{}^{19}C_1x^{18}+\dots$

$(1+x)^{39}=x^{20}({}^{20}C_1.{}^{19}C_0+{}^{20}C_2\,{}^{19}C_1+\dots)$

$\downarrow$

coefficient of x^{20}, $T_{r+1}={}^{39}C_r.x^r$ $r=20$

$\Downarrow$

${}^{39}C_{20}$

So the required value $K=20\times{}^{39}C_{20}$

Now $\frac{((20)!)^2.K}{40!}=\frac{(20!)^2}{40!}\times20\times\frac{39!}{20!\,19!}$

$=10$

$\boxed{\text{Option D}}$

ID-129 Let $y = y(x)$ be the solution of differential eq^n $x\,dy + y\,dx = xy^2\,dx$ which passes through $(1,1)$. Then $y(e^{\pi})$ is

(a) $\frac{e^{\pi}}{1+\pi}$ (b) $\frac{e^{\pi}}{1-\pi}$ (c) $\frac{e^{\pi}}{1+\pi}$ (d) $\frac{e^{\pi}}{1-\pi}$

Sol^n Dividing by xy^2dx we get $\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{y} = 1$

Let $\frac{1}{y} = t \Rightarrow$ differentiating wrt x, $-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$-\frac{dt}{dx} + \frac{t}{x} = 1 \Rightarrow \frac{dt}{dx} - \frac{t}{x} = -1$ (It's a linear differential eq^n)

$IF = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Sol^n. $\frac{1}{x}t = \int \frac{1}{x}\cdot(-1)dx + c \Rightarrow \boxed{\frac{1}{xy} = -\log x + \log c}$

Now $x=1, y=1 \Rightarrow \frac{1}{1} = \log c \Rightarrow c = e$

$\frac{1}{xy} = \log\left(\frac{e}{x}\right)$ When $x = e^{\pi} \Rightarrow \frac{1}{e^{\pi}\cdot y} = -\log e^{\pi} + \log e$

$\frac{1}{e^{\pi}y} = -\pi + 1 \Rightarrow \boxed{y = \frac{e^{-\pi}}{1-\pi}}$ $\boxed{\text{option-B}}$

ID: 1210 Let $f : [-2a, 2a] \to R$ be a thrice differentiable function and g be defined as $g(x) = f(a+x) + f(a-x)$. If 'm' is the minimum no. of roots of $g'(x) = 0$ in the interval $(-a, a)$ and 'n' be minimum no. of roots of $g'''(x) = 0$ in the interval $(-a, a)$ then $(m+n)$ is equal to

(a) 1 (b) 2 (c) 4 (d) 5

Sol^n. Using Rolle's theorem

As $f(x)$ is differentiable function therefore, $g(x)$ is conti/diff in $(-a, a)$

Also $g(a) = g(-a) = f(2a)$

Hence Rolle's theorem is applicable and $g'(x) = 0$ for some x, so $g'(x)$ has at least one root in $(-a, a) \Rightarrow \underline{m = 1}$

$g'(x) = f'(a+x) - f'(a-x)$

$g''(x) = f''(a+x) + f''(a-x)$

Now $g''(a) = f''(2a)$ and $g''(-a) = f''(2a)$

Hence Rolle's theorem is applicable for $g''(x)$ in $(-a, a)$ and $g'''(x)$ has at least 1 root in $(-a, a) \Rightarrow n = 1$

$m + n = 1 + 1 = 2$ $\boxed{\text{Option B}}$

1D-1211 $y = y(x)$ be the solution of the initial value problem $2x\frac{dy}{dx} = 3xe^{y/x} + 2y$, $y(1) = \log_e 3$. Then $y(\frac{1}{e})$ is equal to

(a) $-\frac{1}{e}\log\left(\frac{11}{6}\right)$ (b) $\frac{1}{e}\log\left(\frac{11}{6}\right)$ (c) $-\frac{3}{e}\log\left(\frac{11}{6}\right)$ (d) $\frac{3}{e}\log\left(\frac{11}{6}\right)$

Sol^n. let $y = vx \Rightarrow \frac{dy}{dx} = v.1 + x\frac{dv}{dx}$

$2x\left(v + x\frac{dv}{dx}\right) = 3xe^{v} + 2vx$

$e^{-v}\,dv = \frac{3}{2}\frac{dx}{x} \Rightarrow$ On Integration $-e^{-v} = \frac{3}{2}\log x + C$

$\boxed{-e^{-y/x} = \frac{3}{2}\log x + C}$ Now $x = 1$, $y = \log_e 3$

$-e^{-\log 3} = 0 + C \Rightarrow C = -\frac{1}{3}$

$-e^{-y/x} = \frac{3}{2}\log x - \frac{1}{3}$ sub. $x = \frac{1}{e} \Rightarrow -e^{-ye} = -\frac{3}{2} - \frac{1}{3} = -\frac{11}{6}$

$e^{-ye} = \frac{11}{6} \Rightarrow \log e^{-ye} = \log\left(\frac{11}{6}\right) \Rightarrow -ye = \log\left(\frac{11}{6}\right)$

$ye = -\log\left(\frac{11}{6}\right) \Rightarrow y = -\frac{1}{e}\log\left(\frac{11}{6}\right)$ $\boxed{\text{Option - A}}$

1D-1212 Let $f(t) = \int_0^t e^{x^2}\left((1+2x^2)\sin x + x\cos x\right)dx$. Then the value of $f(\pi) - f(\pi/2)$ is equal to

(a) $-\pi e^{\pi^2/4}$ (b) $-\frac{\pi}{2}e^{\pi^2/4}$ (c) $\frac{\pi}{2}e^{\pi^2/4}$ (d) $\pi e^{\pi^2/4}$

Sol^n let $x^2 = y \Rightarrow 2x\,dx = dy \Rightarrow x\,dx = \frac{dy}{2} \Rightarrow dx = \frac{dy}{2\sqrt{y}}$

$$\int e^y \left[(1+2y)\sin\sqrt{y} + \sqrt{y}\cos\sqrt{y}\right)\frac{dy}{2\sqrt{y}}$$

$$= \frac{1}{2}\int e^y \Big[\underbrace{2\sqrt{y}\sin\sqrt{y}}_{f(y)} + \underbrace{\left(\frac{1}{\sqrt{y}}\sin\sqrt{y} + \cos\sqrt{y}\right)}_{f'(y)}\Big]dy$$

using $\left[\int e^x(f(x) + f'(x))dx = e^x f(x) + c\right]$

$$= \cancel{\frac{1}{2}} \cdot e^y\, \cancel{2}\sqrt{y}\sin\sqrt{y} + c$$

$$f(x) = e^{x^2} \cdot x\sin x + c$$

Now $f(\pi) - f(\pi/2) = 0 - \frac{\pi}{2} \cdot e^{\pi^2/4}$ $\boxed{\text{option-B}}$

ID: 1213 Let $f : [-2, 2] \to R$ be defined by $f(x) = x\sqrt{4-x^2}$. Then which of the following is NOT true

(a) f has two critical points in $(-2, 2)$

(b) Minimum value of f is -2

(c) $x = -2$ is a local minima

(d) f is increasing in $(-\sqrt{2}, \sqrt{2})$

Sol^n.

$$f(x) = x\sqrt{4-x^2}$$

$$f'(x) = x \cdot \frac{-2x}{2\sqrt{4-x^2}} + \sqrt{4-x^2} \Rightarrow f'(x) = \frac{4-2x^2}{\sqrt{4-x^2}}$$

$f'(x) = 0 \Rightarrow x = \pm\sqrt{2}$ (Two critical points)

dec | Inc. | dec — $-\sqrt{2}$, $\sqrt{2}$

Point of minima $= -\sqrt{2}$

Point of maxima $= +\sqrt{2}$

$f(-\sqrt{2}) = -\sqrt{2}\sqrt{4-2} = -2$ (Minimum value)

$\boxed{\text{Option-C}}$

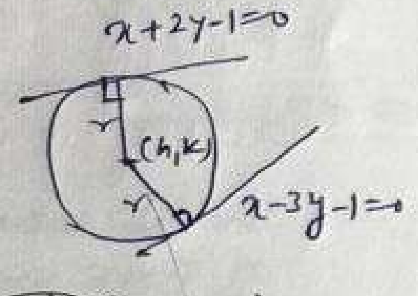
ID: 1214

If the lines $x+2y=1$ and $x-3y=1$ are tangents to a circle, then its centre will lie on

(a) $2x-y=1$ (b) $2x-y=2$ (c) $x^2-y^2-14y-2x+14xy+1=0$

(d) $x^2+y^2+14x-2x-14xy+1=0$

Soln Perpendicular from the centre of circle to the two st. line will be equal to radius



$$\left|\frac{h+2k-1}{\sqrt{h^2+k^2}}\right| = \left|\frac{h-3k-1}{\sqrt{h^2+k^2}}\right|$$

squaring both the sides and replacing (h,k) by (x,y) gives

$x^2+y^2+14xy+14y-2x+1=0$ $\boxed{\text{option-C}}$

ID: 1215

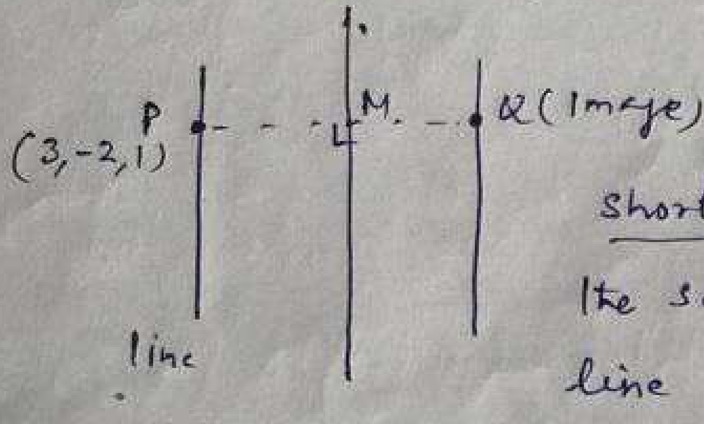
The mirror image of the line $\frac{x-3}{-1}=\frac{y+2}{1}=\frac{z-1}{1}$ with respect to the plane $3x-y+4z=2$ is

(a) $\frac{x}{-1}=\frac{y+1}{1}=\frac{z+3}{1}$ (b) $\frac{x}{1}=\frac{y+1}{1}=\frac{z+3}{1}$

(c) $\frac{x+1}{-1}=\frac{y}{-1}=\frac{z+2}{1}$ (d) $\frac{x+1}{-1}=\frac{y}{-1}=\frac{z+2}{-1}$

Soln. dr of line & plane are $\langle -1,1,1\rangle$ & $\langle 3,-1,4\rangle \Rightarrow -3-1+4=0$

ie line and plane are parallel



shortcut: only option (a) has the same dr's as that of given line

$\boxed{\text{option-A}}$

ID: 1216 Let $\hat{a}$ and $\hat{c}$ be collinear vectors such that $(\bar{b}-4\hat{c})=-9\hat{a}$ 9.
for a vector $\bar{b}$. Then $|\bar{b}|^2$ is equal to
(a) 27 (b) 25 (c) 21 (d) 18

Sol^n. $\bar{b}-4\hat{c}=-9\hat{a} \Rightarrow \bar{b}=-9\hat{a}+4\hat{c}$

$|\bar{b}|^2=(-9\hat{a}+4\hat{c}).(-9\hat{a}+4\hat{c})$

$|\bar{b}|^2= 81|\hat{a}|^2+16|\hat{c}|^2-72\hat{a}.\hat{c}$ ($\hat{a}$ & $\hat{c}$ are collinear $\theta=0°$)

$|\bar{b}|^2 = 81+16-72|\hat{a}||\hat{c}|\cos 0° = 81+16-72 = \underline{27}$ $\boxed{\text{option: a}}$

ID: 1217
The probability that two randomly selected distinct 2-digit natural numbers have a common factor either 2 or 3 is:
(a) $\frac{88}{267}$ (b) $\frac{95}{267}$ (c) $\frac{1}{3}$ (d) $\frac{608}{1617}$

Sol^n. Two digit no. multiple of 2 $(10, 12, 14 \ldots 98) = 45$
Two digit no. multiple of 3 $(12, 15, 18 \ldots 99) = 30$
Two digit no multiple of 6 $(12, 18, \ldots 96) = 15$

Required Prob $= \dfrac{{}^{45}C_2 + {}^{30}C_2 - {}^{15}C_2}{{}^{90}C_2} = \dfrac{88}{267}$ $\boxed{\text{option: a}}$

ID. 1218. The value of $\int_{-1}^{2} |x^3 \sin \pi x|\, dx$ is equal to
(a) $\frac{11}{\pi}-\frac{4}{\pi^2}-\frac{6}{\pi^3}$ (b) $\frac{11}{\pi}-\frac{30}{\pi^3}$ (c) $\frac{11}{\pi}+\frac{4}{\pi^2}-\frac{6}{\pi^3}$ (d) $\frac{11}{\pi}+\frac{30}{\pi^3}$

Sol^n.

x:	-1	$-1/2$	0	$1/2$	1	$3/2$	2
x^3:	-1	$-1/8$	0	$1/8$	1	$27/4$	8
πx:	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\sin \pi x$:	$-ve$			$+ve$		$-ve$	
$x^3 \sin \pi x$:	$+ve$			$+ve$		$-ve$	

The required integral $= \int_{-1}^{1} x^3 \sin \pi x\, dx + \int_{1}^{2} -x^3 \sin \pi x\, dx$

$= 2\int_0^1 x^3 \sin \pi x\, dx - x^3 \int_1^2 \sin \pi x\, dx$

Now $\int x^3 \sin\pi x\, dx = -\frac{\cos\pi x}{\pi}.x^3 - \int 3x^2.\frac{-\cos\pi x}{\pi}dx$

$= -\frac{x^3\cos\pi x}{\pi} + \frac{3}{\pi}\left[x^2.\frac{\sin\pi x}{\pi} - \int 2x.\frac{\sin\pi x}{\pi}dx\right]$

$= -\frac{x^3\cos\pi x}{\pi} + \frac{3x^2}{\pi^2}.\sin\pi x - \frac{6}{\pi^2}\int x.\sin\pi x\, dx$

$= -\frac{x^3\cos\pi x}{\pi} + \frac{3x^2}{\pi^2}.\sin\pi x - \frac{6}{\pi^2}\left[x.\frac{-\cos\pi x}{\pi} - \int 1.\frac{-\cos\pi x}{\pi}dx\right]$

$= -\frac{x^3\cos\pi x}{\pi} + \frac{3x^2}{\pi^2}.\sin\pi x + \frac{6x}{\pi^3}.\cos\pi x - \frac{6}{\pi^4}\sin\pi x$

On substituting limits we get $\left(\frac{11}{\pi} - \frac{30}{\pi^3}\right)$ $\boxed{\text{option - B}}$

ID:1219 The converse of the logical statement $(p\wedge(\sim q)) \Rightarrow (p \vee q)$ is

(a) p (b) q (c) $\sim p$ (d) $\sim q$

Sol^n.

converse

p	q	$p\vee q$	$\sim q$	$p\wedge\sim q$	$(p\wedge(\sim q))\Rightarrow(p\vee q)$	$(p\vee q)\Rightarrow(p\wedge(\sim q))$
T	T	T	F	F	T	F
T	F	T	T	T	T	T
F	T	T	F	F	T	F
F	F	F	T	F	T	T

$\boxed{\text{option : D}}$

ID: 1220

Consider ellipse E: $\frac{x^2}{9}+\frac{y^2}{4}=1$ and hyperbola H: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with eccentricities e_1, e_2 respectively. If the hyperbola passes through the focus of ellipse E and $e_1 : e_2 = 1:3$ then the length of latus rectum of the hyperbola H is equal to

(a) $2\sqrt{5}$ (b) $4\sqrt{5}$ (c) $8\sqrt{5}$ (d) $10\sqrt{5}$

Sol^n. E: $\frac{x^2}{9}+\frac{y^2}{4}=1$ H: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$e_1 = \sqrt{1-4/9} = \frac{\sqrt{5}}{3}$ $e_2 = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{\frac{a^2+b^2}{a^2}}$

$\frac{e_1}{e_2} = \frac{1}{3} \Rightarrow 9e_1^2 = e_2^2 \Rightarrow 9\times\frac{5}{9} = \frac{a^2+b^2}{a^2} \Rightarrow \boxed{4a^2 = b^2}$ —①

Focus of E $(\pm ae, 0) \Rightarrow (\pm 3\frac{\sqrt5}{3}, 0) = (\pm\sqrt5, 0)$, H passes thro $(\pm\sqrt5, 0)$

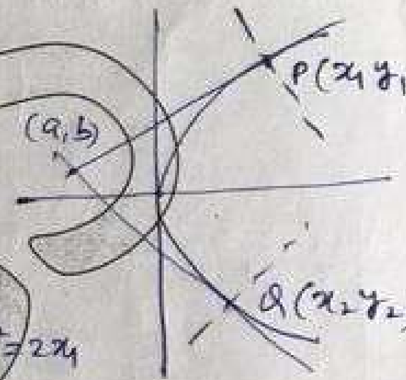
ie $\frac{5}{a^2} - \frac{1}{b^2} = 1$ —② solving ① & ② $b^2 = 20$, $a = \sqrt5$

$LR = \frac{2b^2}{a} = \frac{2\times20}{\sqrt5} = 8\sqrt5$ $\boxed{\text{option: C}}$

<u>ID: 1221</u>

Let $\sqrt3 x + y = \frac{5\sqrt3}{2}$ and $\sqrt5 x + y = \frac{7\sqrt5}{2}$ be two normal lines to the parabola $y^2 = 2x$ at points P & Q. If the tangent lines at P and Q intersect at the point (a, b) then the value of $b^2 - a$ is-----

<u>Soln</u>:



$y^2 = 2x \Rightarrow 2y\frac{dy}{dx} = 2$

$\frac{dy}{dx} = m_T = \frac{1}{y}$

Slope of $\sqrt3 x + y = \frac{5\sqrt3}{2}$ $m_N = -\sqrt3$

$m_T \times m_N = -1 \Rightarrow \frac{1}{y_1}\times -\sqrt3 = -1 \Rightarrow \boxed{y_1 = \sqrt3}$, $y_1^2 = 2x_1$

$P \Rightarrow \boxed{x_1 = 3/2}$

<u>tangent at P</u> $\Rightarrow y - \sqrt3 = \frac{1}{\sqrt3}(x - 3/2) \Rightarrow \boxed{\sqrt3 y - x - 3/2 = 0}$ —(i)

Now Slope of $\sqrt5 x + y = \frac{7\sqrt5}{2} \Rightarrow m_N = -\sqrt5 \Rightarrow m_T = \frac{1}{\sqrt5} = \frac{1}{y_2} \Rightarrow y_2 = \sqrt5$

sub. in $y^2 = 2x$, $x_2 = \frac{5}{2}$

Point Q$(5/2, \sqrt5)$

Tangent at Q $\Rightarrow y - \sqrt5 = \frac{1}{\sqrt5}(x - 5/2) \Rightarrow \boxed{\sqrt5 y - x - 5/2 = 0}$ —(ii)

solving (i) & (ii) $x = \frac{\sqrt{15}}{2}$, $y = \frac{\sqrt5 + \sqrt3}{2}$

$b^2 - a = \left(\frac{\sqrt5+\sqrt3}{2}\right)^2 - \frac{\sqrt{15}}{2} = 2$ $\boxed{\text{Ans: 2}}$

<u>ID: 1222</u> If the normal to the curve $(y - x^5)^2 = x(1+x^2)^2$ at the point $(1, 3)$ passes through the point $(\alpha, 2)$ then $|\alpha|$ is equal to -----.

$(y-x^5)^2 = x(1+x^2)^2$; Differentiating wrt x

$$2(y-x^5)\left(\frac{dy}{dx}-5x^4\right) = 1.(1+x^2)^2 + x.(2(1+x^2).2x)$$

at $(1,3) \Rightarrow 2(3-1)(m_T-5) = (1+1)^2+4(1+1) \Rightarrow m_T = 8, \Rightarrow m_N = -\frac{1}{8}$

Eq^n of normal $y-3 = -\frac{1}{8}(x-1)$ it passes thro $(\alpha, 2)$

$$2-3 = -\frac{1}{8}(\alpha-1) \Rightarrow 8 = \alpha - 1 \Rightarrow \boxed{\alpha = 9}$$

ID:1223 If the system of linear equations $2x-3y+5z=\beta$, $\alpha x+y+2z=3$, $3x-16y+23z=-13$ has infinitely many solutions then $(\alpha+\beta)$ is equal to ____

Soln. For infinitely many solutions, $|A|=0$, $D_1=D_2=D_3=0$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ \alpha & 1 & 2 \\ 3 & -16 & 23 \end{vmatrix} = 0 \Rightarrow \boxed{\alpha=7}$$

Now $$D_1 = \begin{vmatrix} \beta & -3 & 5 \\ 3 & 1 & 2 \\ -13 & -16 & 23 \end{vmatrix} = 0 \Rightarrow \beta(23+32)+3(69+26)+5(-48+13)=0$$

$$\boxed{\beta = -2}$$

$$\boxed{\alpha+\beta = 5}$$

ID:1224 Let $f: N \to N$ be a function defined by $f(n) = an^2+bn+c$, If $f(1)=3$, $f(2)=6$ and $f(n) = \frac{f(n-1)+f(n-2)+8n^2-}{6}$ for every $n \geq 3$ then $f(100)$ is ____

Soln. $n=3$ $\quad f(3) = \frac{f(2)+f(1)+8-3}{6} \Rightarrow f(3)=13$

$n=4$ $\quad f(4) = \frac{f(3)+f(2)+8\times16-3}{6} \Rightarrow f(4) = 24$

$$f(n) = 3+6+13+24+\dots+a_{n-1}+a_n$$

$$f(n) = \quad 3+6+13+\dots+a_{n-1}+a_n$$

$$0 = 3 + \underbrace{[3+7+11+\dots]}_{AP,\ (n-1)\text{ terms}} - a_n$$

$$a_n = 3 + \frac{n-1}{2}(6+(n-2)\times4) \Rightarrow f(n) = 2n^2-3n+4$$

$$f(100) = 2(100)^2 - 300 + 4 = \underline{19,704}$$

ID:1225

If the coefficient of x^8 in the expansion of $(1-x^2)^3(1+2x^3)^7(1+x^4)^5$ is β then $|\beta|$ is equal to ____

Soln.
$$(1-x^2)^3(1+2x^3)^7(1+x^4)^5$$
$$(1-3x^2+3x^4+x^6)\left[1+14x^3+\frac{7\times6}{2}.4x^6+\cdots\right]\left[1+5x^4+10x^8+\cdots\right]$$
$$(1-3x^2+3x^4+x^6)\left[1+5x^4+10x^8+14x^3+70x^7+84x^6+\cdots\right]$$
$$=10x^8-252x^8+15x^8=-227x^8 \qquad \boxed{-227}$$

ID:1226

If for real numbers α, β, $\int\frac{1+x\cos x}{x(1-x^2e^{2\sin x})}dx = \alpha\log_e\left|\frac{1}{x^2e^{2\sin x}}-\beta\right| +$ constant the the value of $10(\alpha+\beta)$ is ____

Soln. On differentiation

$$\frac{1+x\cos x}{x(1-x^2e^{2\sin x})}=\frac{\alpha}{\left|\frac{1}{x^2e^{2\sin x}}-\beta\right|}\cdot\frac{-1}{(x^2e^{2\sin x})^2}\times\left[2xe^{2\sin x}+x^2e^{2\sin x}.2\cos x\right]$$

$$\frac{1+x\cos x}{x(1-x^2e^{2\sin x})}=\frac{-2\alpha(1+x\cos x)}{(1-\beta.x^2e^{2\sin x})}$$

On comparing we get

$$-2\alpha=1 \Rightarrow \alpha=-1/2,\ \beta=1 \Rightarrow \alpha+\beta=\frac{1}{2} \Rightarrow 10(\alpha+\beta)=5$$

ID:1227

If the mean and variance of the observations 2, 6, α, 10, 12, β, 15 are 9 and 18 respectively then $\alpha\beta$ is ____

Soln.
$$\bar{x}=9 \Rightarrow \frac{2+6+\alpha+10+12+\beta+15}{7}=9 \Rightarrow \alpha+\beta=18$$
$$\sigma^2=18 \Rightarrow 18=\frac{(2-9)^2+(6-9)^2+(\alpha-9)^2+(10-9)^2+(12-9)^2+(\beta-9)^2+(15-9)^2}{7}$$
$$126=49+9+(\alpha-9)^2+1+9+(\beta-9)^2+36$$

$22 = (\alpha-9)^2+(\beta-9)^2 \Rightarrow 22 = \alpha^2+\beta^2-18(\alpha+\beta)+162$

$0 = (\alpha+\beta)^2-2\alpha\beta-18(\alpha+\beta)+140 \Rightarrow 0 = 18^2-2\alpha\beta-18\times18+140$

$\boxed{\alpha\beta = 70}$

ID: 1228

The number of real solutions of $e^{4x}+4e^{3x}-e^{2x}-10e^x+6=0$ is equal to _____

Sol^n: Let $y=e^x$ then $y^4+4y^3-y^2-10y+6=0$

When $y=1$ we get $1+4-1-10+6=0$ so $e^x=1 \Rightarrow \underline{x=0}$ is one solution.

If we divide $y^4+4y^3-y^2-10y+6=0$ by $(y-1)$ we get

let $f = y^3+5y^2+4y-6=0$ $(y=e^x)$

$f' = 3y^2+10y+4 > 0$ (i.e. f is increasing)

$f(\frac{1}{2}) < 0$ and $f(1) > 0$ so f will intersect x-axis exactly at $\underline{1 \text{ point}}$. $\Rightarrow$ Hence there will be $\underline{2 \text{ Sol}^n}$.

ID. 1229

Let $A_1, A_2, A_3 \ldots$ be an increasing GP of positive real numbers. If $A_6 = 49A_2$ and $A_6+A_3A_5 = 8$, then $A_7(A_1+A_3) =$ _____

Sol^n. $A_6 = 49A_2 \Rightarrow ar^5 = 49ar \Rightarrow \underline{r^4=49}$, $\underline{r^2=7}$

$A_6+A_3A_5 = 8 \Rightarrow ar^5+ar^2\cdot ar^4 = 8 \Rightarrow ar^5(1+ar) = 8$

$ar\cdot r^4(1+ar) = 8 \Rightarrow 49y(1+y) = 8$ (let $y=ar$)

$49y^2+49y-8=0 \Rightarrow 49y^2+56y-7y-8=0 \Rightarrow y=\frac{1}{7}, y=-\frac{8}{7}$ $(\times)$

So, $\underline{ar=\frac{1}{7}}$

Now, $A_7(A_1+A_3) = ar^6(a+ar^2) = a^2r^6(1+r^2)$

$= (ar)^2 r^4(1+r^2) = \frac{1}{49}\times49(1+7) = 8$ $\boxed{8}$

ID 1230

suppose that $\bar{a}, \bar{b}$ and $\bar{c}$ are non-coplanar vectors in R^3. let the components of a vector along $\bar{a}, \bar{b}$ and $\bar{c}$ be 2, 5 and 3 respectively. If the components of this vector $\bar{n}$ along $\bar{a}+2\bar{b}-c$, $-2\bar{a}+\bar{b}+\bar{c}$ and $\bar{a}-\bar{b}-2\bar{c}$ are x, y and z respectively then the value of $x+y-4z$ is equal to ___

Soln Take $\bar{a}, \bar{b}, \bar{c}$ and i, j and k and solve.

$$\frac{\bar{n}.\bar{a}}{|a|} = \frac{n.i}{|i|} = 2 \Rightarrow n.i = 2$$

$$\frac{n.\bar{b}}{|b|} = \frac{n.j}{|j|} = 5$$

$$\frac{n.\bar{c}}{|\bar{c}|} = \frac{n.k}{|k|} = 3$$